**Math 120  
1.8 Inverse Functions**

# Objectives

1. Verify inverse functions.
2. Find the inverse of a function.
3. Use the graph of a one-to-one function to graph its inverse function.
4. Find the inverse of a function and graph both functions on the same axes.

# Topic #1: The Inverse of a Function

**In an inverse relationship, the domain and range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

However, inverting a function **does not guarantee** the inverse relation is also a function.

One definition of an inverse is that all members in the domain interchange with their associated member in the range. In other words, **each ordered pair in the function becomes \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ in the inverse.**

Consider the two functions:

Suppose **function** consists of the ordered pairs (this is a function since no x-values repeat):

The inverse of function consists of the ordered pairs:

Is the inverse of function *f* a function? Why or why not?

It is worth noting that function f has no repeated x-values ***and*** no repeated y-values. This is the definition of a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** (abbreviated as 1:1), which tells us the function MUST have an inverse function.

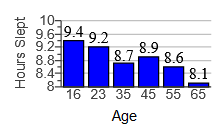
Suppose **function**  consists of the ordered pairs (this is also a function since no x-values repeat):

The inverse of function consists of the ordered pairs:

Is the inverse of function a function? Why or why not?

Notice function g has repeated y-values, which results in the inverse relation for function g having repeated x values! (Because the inverse interchanges the x’s and y’s). This tells us that function *g* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ have an inverse function.

*Example #1* – Determine if the Function has an Inverse: the graph below shows the average hours slept for select age groups:



Let x be:

Let f(x) be:

a) Write the ordered pairs of the function (this is a function because no x’s repeat).

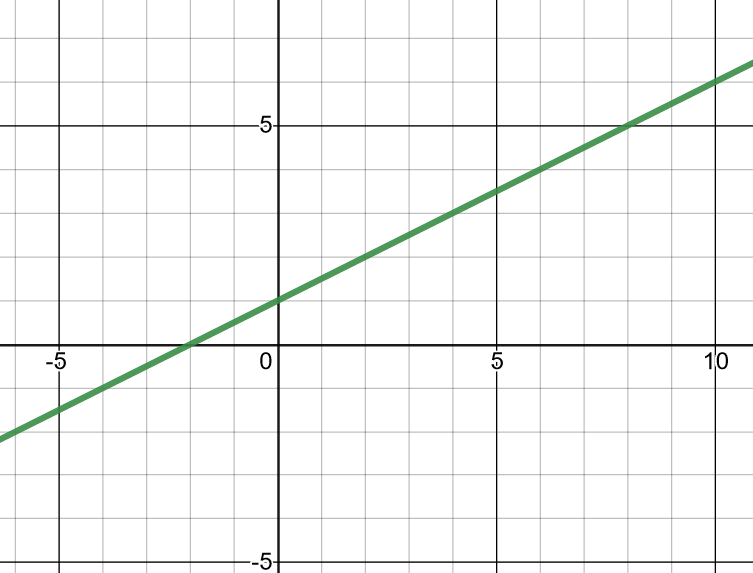
The ordered pairs are:

b) Does the function have an inverse that is also a function? Explain

By definition of inverse, interchange all members of the domain with each member in the range:

*Example #2* – Graph the Inverse for the Function

A graph of is given, graph its inverse.



Note that if you are given the graph of **any** 1:1 function, you can \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_to sketch the graph of the inverse function.

This is linear, so its inverse must also be a line. All we need are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ We can pick any 2 points and interchange them to sketch the graph of the inverse function:

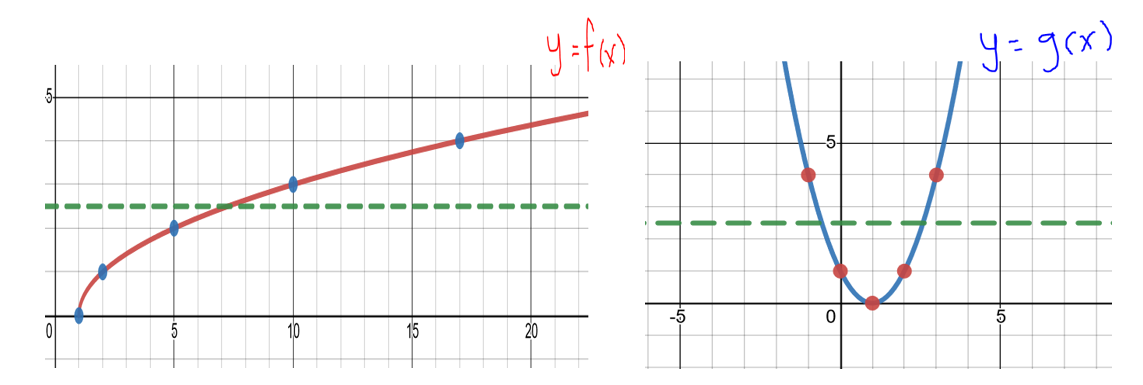


Note the symmetry across the line y=x!

# Topic #2: One-to-One Functions and the Horizontal Line Test

Functions that are one-to-one (abbreviated as 1:1) have an inverse. Functions that are not 1:1 do not have an inverse.

A graph of a function tells us if it is 1:1 or not. If a horizontal line does not intersect the graph more than once, then it is 1:1 since y does not repeat. Consider the graphs of two functions:



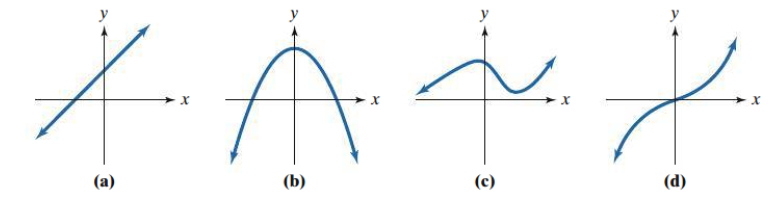
Function passes the Horizontal Line Test and is 1:1. As a result, function \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Function fails the Horizontal Line Test and is not 1:1. As a result, function \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*YOU TRY #1* – Determine if the Function has an Inverse

The graphs of 4 functions follow.

Which functions have an inverse? Which functions do not have an inverse? Explain your reasoning.



**Formal definition of Inverse function (using function composition):**

Let *f* and *g* be two functions such that

for every x in the domain of g and

for every x in the domain of f.

The function, *g*, is the **inverse of the function *f***and is denoted by \_\_\_\_\_\_\_\_\_\_\_\_\_

(read as “f inverse”; the -1 is NOT an exponent!).

Thus,  =\_\_\_\_\_ and\_\_\_\_\_\_\_\_\_\_\_\_\_*.*

The domain of *f* is equal to the \_\_\_\_\_\_\_\_\_ of , and vice versa.

*Example #1* - and,

SHOW they are inverse functions using the formal definition above:

*Example #2* - and,

SHOW they are inverse functions using the formal definition above:

*YOU TRY #2* -

 and ;

SHOW they are inverse functions:

# Topic #3: Finding the Inverse of a 1:1 Function

When a function is 1:1 it has an inverse. If function is 1:1 and contains the points , then its inverse contains the points .

In other words, the **inputs** trade places with the **\_\_\_\_\_\_\_\_\_\_\_\_\_.**

The domain and the range are interchanged; **the domain for is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ for and the range for is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ for**

**To find the inverse for a function:**

1. Replace with y and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the variables and ;
2. Solve for\_\_\_\_\_\_\_\_ and the result is the inverse for the original function (and the original function is the inverse for the new function);
3. Last, we replace y in the equation for the inverse function with function notation \_\_\_\_\_\_\_\_\_\_\_\_\_

*Example #1* - Consider the function .

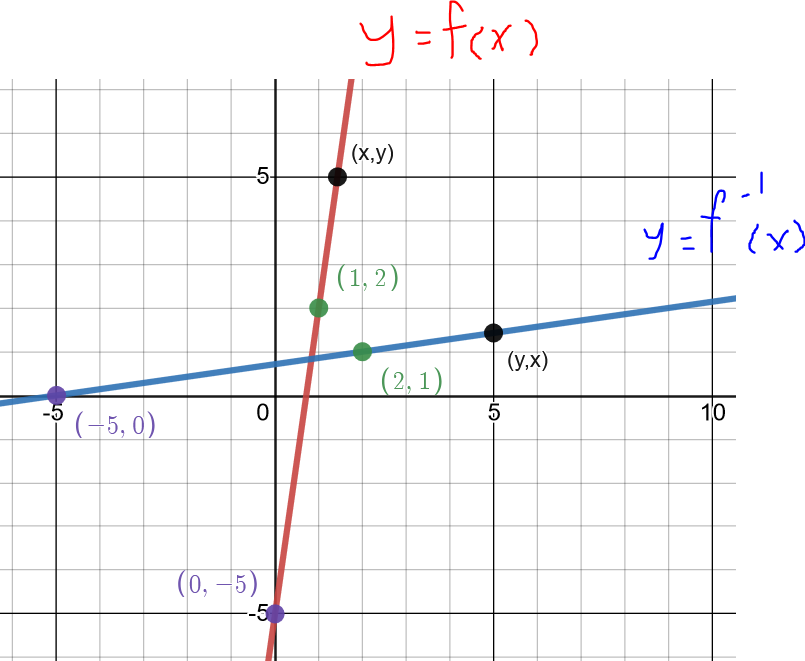
To find the inverse function we rewrite without function notation, replacing with y:

Step 1: we interchange and :

Step 2: we solve for (this will create a new function where is a function of ).

Step 3: Last, we identify the inverse with the notation:

A graph shows, the 2 functions have all x and y values interchanged.



Another way to look at this and check visually is that ***the graph of the inverse is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of the graph of over the line y=x***

***This is true for every function and its inverse function.***

A general graph showing the inverse relationship is shown below

Figure is labeled 1.67, The graph of f inverse is a reflection of the graph of f about _______.
(x,y) axis with a blue line, f(x), beginning a third of the way up the y axis from 0, curving upward, labeled graph of f. Green line, f inverse, beginning a third of the way right of the x axis from 0, curving upward and right as a mirror image of the blue line. Dotted line of y=x in the middle, point (a,b) on blue line, point (b,a) on green line.


It is worth pointing out that the operations on the 2 functions in the above example are completely OPPOSITE and IN REVERSE:

starts by **multiplying** by 7, then **subtracting** 5, and starts by **adding** 5, then **dividing** by 7.

*Example #2* – Find the Inverse for the Function

a)

First, rewrite the function using y in place of *f(x)*:

Next, interchange x and y (which means to apply the definition of inverse):

Solve for (this is the hardest step):

The last step is to indicate with notation that the above equation is the inverse of :

Notice the operations are OPPOSITES and REVERSED.

The domain for the original function is \_\_\_\_\_\_\_\_\_\_ and the range is\_\_\_\_\_\_\_\_\_\_\_ The inverse function uses the range of the original function for its domain \_\_\_\_\_\_\_\_\_\_and uses the domain for its range \_\_\_\_\_\_\_\_\_\_\_\_\_

b)

First, rewrite the function using y in place of *f(x)*:

Next, interchange x and y (apply the definition of inverse):

Solve for :

The last step is to indicate with notation that the above equation is the inverse of :

Notice the operations are OPPOSITES and REVERSED.

The domain for the original function is \_\_\_\_\_\_\_\_\_\_\_ and the range is\_\_\_\_\_\_\_\_\_. The inverse function uses the range of the original function for its domain and uses the domain for its range \_\_\_\_\_\_\_\_\_\_\_\_\_\_

c)

Note the domain is \_\_\_\_\_\_\_\_\_ and the range is \_\_\_\_\_\_\_

If you aren’t sure, graph it in your calculator.

First, rewrite the function using y in place of *f(x)*:

Next, interchange x and y (apply the definition of inverse):

Solve for :

The new function has a domain \_\_\_\_\_\_\_\_\_\_\_\_and a range \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Had the domain of not been restricted, then the new function would not be 1:1 and would not be an inverse!

d)

This function is restricted to make it 1:1.

Note the restricted domain is \_\_\_\_\_\_\_\_\_ and the restricted range is \_\_\_\_\_\_\_

If you aren’t sure, graph it in your calculator.

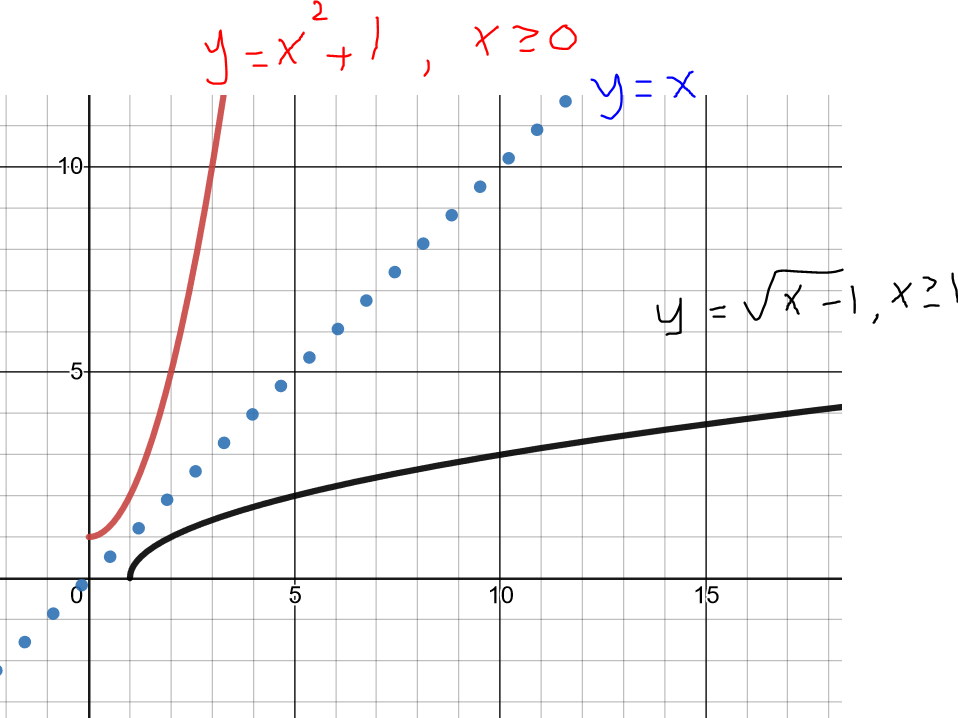
First, rewrite the function using y in place of *f(x)*:

Next, interchange x and y (apply the definition of inverse):

Solve for :

The new function has a domain \_\_\_\_\_\_\_\_\_\_\_\_and a range \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

A graph of the original functions from example c) and d) show that if function 1 is the inverse of function 2, then function 2 must be the inverse of function 1. Also, notice the symmetry about the line .



*YOU TRY #4* – Find the equation for 

Use interval notation to give the domain and range of *f* and *f-1(x).*